

CP Violation and Weak Decays

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Abstract

I review some of the salient issues connected to CP-violation and weak decays. In particular, I focus on recent and forthcoming tests of the CKM model and discuss the accuracy one can expect to achieve in B-decays for the parameters of this model.

1 Introduction

There is a large difference in emphasis between the study of weak decays and decays which lead to CP-violating effects. The former, paradoxically, essentially test our understanding of the strong interactions, while CP-violating phenomena explore the limits of our knowledge. Of course, in practice, both phenomena are linked and the extraction of CP-violating effects is often clouded by our imperfect understanding of the strong interactions. We will see a case in point below when we discuss ϵ'/ϵ .

In view of these considerations, it has proven natural to focus one's attention in systems like B-decays where one can best control the effects of the strong interactions. Indeed, matters simplify considerably if one looks at heavy quark systems where $m_Q \gg \Lambda_{\text{QCD}}$. This can perhaps best be appreciated by considering the ratio of charged to neutral lifetimes in heavy-light $q\bar{Q}$ bound states. For Kaons, where $m_Q \equiv m_s \sim \Lambda_{\text{QCD}}$, this ratio is nearly 140. For D mesons, where $m_Q \equiv m_c \sim (5-10)\Lambda_{\text{QCD}}$, this ratio is reduced to about 2.5. However, for B mesons, where $m_Q \equiv m_b \sim (15-30)\Lambda_{\text{QCD}}$, this ratio is very close to unity: $\tau(B^+)/\tau(B^0) = 1.072 \pm 0.026$.¹ Although one has some understanding why these τ_+/τ_0 enhancements occur,^{2,3} clearly life is simpler for B decays. Here, to a first approximation, one can neglect the effects of the light-quark spectator, so that B -decays are essentially b -decays. Furthermore, for B -decays one can use a systematic expansion in the heavy quark mass m_b —the, so called, Heavy Quark Effective Theory (HQET)⁴—to effectively incorporate corrections of $O(\Lambda_{\text{QCD}}/m_b)$ to the simple spectator approximation.

2 CP Violation—Preliminaries

Although the study of CP violation is a mature subject, we still have very limited experimental information. This consists of:

- i) Measurements of certain CP-violation parameters in the $K^0 - \bar{K}^0$ complex. Until recently, all these measurements could be explained in terms of the $\Delta S = 2$ complex parameters ϵ . However, very recently, new evidence for a non-vanishing value for the $\Delta S = 1$ parameter ϵ' ⁵ has been announced.
- ii) Bounds on other CP-violating or T-violating quantities. Prototypical of these are the strong bound on the electric dipole moment of the neutron [$d_n \leq 1.2 \times 10^{-25}$ e cm (95% C.L.)¹].
- iii) Indirect evidence from cosmology. Here the strongest evidence is the observed baryon—antibaryon asymmetry of the Universe, embodied in the ratio $\eta_B = (n_B - n_{\bar{B}})/n_\gamma$ which is of order $\eta_B \sim (3 - 4) \times 10^{-10}$.⁶

Theoretically, one is not much better off. Although we understand the framework needed to have CP-violation in a theory, we really still do not understand the details of how CP-violation occurs in nature. In particular, even though we have a working paradigm for CP-violation, the CKM⁷ model, we actually have no real proof of the validity of this paradigm. In fact, we have indirect evidence that the CKM paradigm must fail at some level!

It is useful to describe the present theoretical prejudices regarding CP-violation. The first of these is that the observed CP-violation is due to an explicit breaking of CP at the Lagrangian level, and not as a result of spontaneous CP-violation. This latter possibility is disfavored by cosmology, as it leads to too much energy density in the domain walls separating different CP-domains in the Universe.⁸ The second prejudice is that CP violation is connected with renormalizable interactions. If CP is violated, then all parameters which can be complex in the Lagrangian must be included. For example, in a two-Higgs model, it is inconsistent to have complex Yukawa couplings of the Higgs bosons to fermions and a real Higgs-Higgs coupling μ^2 . Fermionic loops will induce (infinite) complex contributions to μ^2 and these can only be absorbed if μ^2 itself is taken as a complex parameter. So, renormalizability requires μ^2 to be complex.

The upshot of these considerations is that, in general, the number of CP-violating phases entering into a theory increases with the complexity of the

theory. In this respect, the three-generations CKM model⁷ is the simplest possible example of a CP-violating theory. In the CKM model there are two CP-violating phases. One of these is the CP-violating phase γ (arising from the Yukawa interactions) which enters in the quark mixing matrix V_{CKM} . The other phase is the CP-violating vacuum angle $\bar{\theta}$ which accompanies the CP-odd gluon density $F\tilde{F}$.⁹ Even in this most simple of models, however, we do not really understand why these two CP-violating phases are so vastly different. While $\gamma \sim O(1)$, the parameter $\bar{\theta} < 10^{-10}$, so as not to obtain too large a dipole moment for the neutron. This is the strong CP-problem.

If one ignores the strong CP problem, then it appears that the CKM model gives a simple and consistent description of all existing experimental data. As we mentioned earlier, the CP-violating phenomena in the neutral Kaon system are connected to the ϵ parameter, which measures the CP even admixture in the K_L state:

$$|K_L\rangle = \frac{1}{\sqrt{2}}(|K^0\rangle - |\bar{K}^0\rangle) + \epsilon \frac{1}{\sqrt{2}}(|K^0\rangle + |\bar{K}^0\rangle) = |K_2\rangle + \epsilon|K_1\rangle. \quad (1)$$

This parameter is experimentally small $|\epsilon| \simeq |\eta_{00}| \simeq |\eta_{+-}| \sim 2 \times 10^{-3}$.¹ In the CKM model this smallness is understood as resulting from the smallness of the interfamily mixings, not because γ is small. One finds: $\epsilon \sim \lambda^4 \sin \gamma \sim 10^{-3} \sin \gamma$, where λ is the sine of the Cabibbo mixing angle, $\lambda \sim 0.22$.

Nevertheless, the observed matter-antimatter asymmetry in the Universe suggests that there are other significant CP violating phases, besides the CKM phase γ . As is well known,¹⁰ to establish a non-zero asymmetry η_B one needs to have B- and CP-violating processes go out of equilibrium during the evolution of the Universe. This can occur within the framework of grand unified theories (GUTs), but could also have happened at the time of the electroweak phase transition, if this transition was strongly first-order. In either case, it is easy to establish that η_B necessarily depends on other phases besides γ . This is clear in the case of GUTs, since these theories involve further interactions beyond those of the standard model. For electroweak baryogenesis the argument is more subtle. It turns out that if γ is the only phase present at the electroweak phase transition then, because of the GIM mechanism¹¹, η_B is very small. Typically, one finds¹²

$$\eta_B \sim \epsilon_{\text{CP viol}} \sim \lambda^6 \sin \gamma \prod_{i < j} \frac{(m_j^2 - m_i^2)}{(M_W)^{12}} \sim 10^{-18} \sin \gamma, \quad (2)$$

so the generated asymmetry is negligible. Furthermore, if there is only one Higgs doublet, given the present bound on $M_H \geq 90$ GeV from LEP200,¹³

the electroweak phase transition is only weakly first order and even the tiny asymmetry established gets erased. Both these difficulties, in principle, can be obviated in multi-Higgs models. However, these models introduce other CP-violating phases besides γ .

3 CP-Violation—Testing the CKM Paradigm.

The consistency of the CKM model with the observed CP-violating phenomena in the Kaon system emerges from a careful study of constraints on the CKM mixing matrix. It is useful for these purposes, following Wolfenstein,¹⁴ to expand the elements of V_{CKM} in powers of the Cabibbo angle $\lambda = \sin \theta_c = 0.22$:

$$V_{\text{CKM}} \simeq \begin{pmatrix} 1 - \lambda^2/2 & \lambda & A\lambda^3(\rho - i\eta) \\ -\lambda & 1 - \lambda^2/2 & A\lambda^2 \\ A\lambda^3(1 - \rho - i\eta) & -A\lambda^2 & 1 \end{pmatrix} + O(\lambda^4) . \quad (3)$$

One sees from the above that, to $O(\lambda^4)$, the only complex phases in V_{CKM} enter in the V_{ub} and V_{td} matrix elements:

$$V_{ub} = A\lambda^3(\rho - i\eta) \equiv |V_{ub}|e^{-i\gamma}; \quad V_{td} = A\lambda^3(1 - \rho - i\eta) \equiv |V_{td}|e^{-i\beta} . \quad (4)$$

The unitarity condition $\sum_i V_{ib}^* V_{id} = 0$ on the V_{CKM} matrix elements has a nice geometrical interpretation in terms of a triangle in the $\rho - \eta$ plane with base $0 \leq \rho \leq 1$ and with an apex subtending an angle α , where $\alpha + \beta + \gamma = \pi$.

One can use experimental information on $|\epsilon|$, the $B_d - \bar{B}_d$ mass difference, Δm_d , and the ratio of $|V_{ub}|/|V_{cb}|$ inferred from B -decays to deduce a 95% C. L. allowed region in the $\rho - \eta$ plane. If one includes, additionally, information from the recently obtained strong bound on $B_s - \bar{B}_s$ mixing [$\Delta m_s > 12.4 \text{ ps}^{-1}$ (95% C.L.)¹⁵], one further restricts the CKM allowed region. Fig. 1 shows the result of a recent study for the Babar Physics Book.¹⁶ As one can see, the data is consistent with a rather large CKM phase γ : $45^\circ \leq \gamma \leq 120^\circ$. If one were to imagine that $|\epsilon|$ is due to some other physics, as in the superweak theory,¹⁷ then effectively the $\Delta S = 1$ parameter $\eta \simeq \gamma \simeq 0$. In this case one has another allowed region for ρ at the 95% C.L.: $0.25 \leq \rho \leq 0.27$.^a

The only CP violating constraint in Fig. 1 is provided by $|\epsilon|$. So, to really prove that the CKM phase γ is large, one needs to measure another

^aAs we will discuss below, the non-zero result for ϵ'/ϵ recently announced by the KTeV Collaboration,⁵ by itself excludes this superweak option.

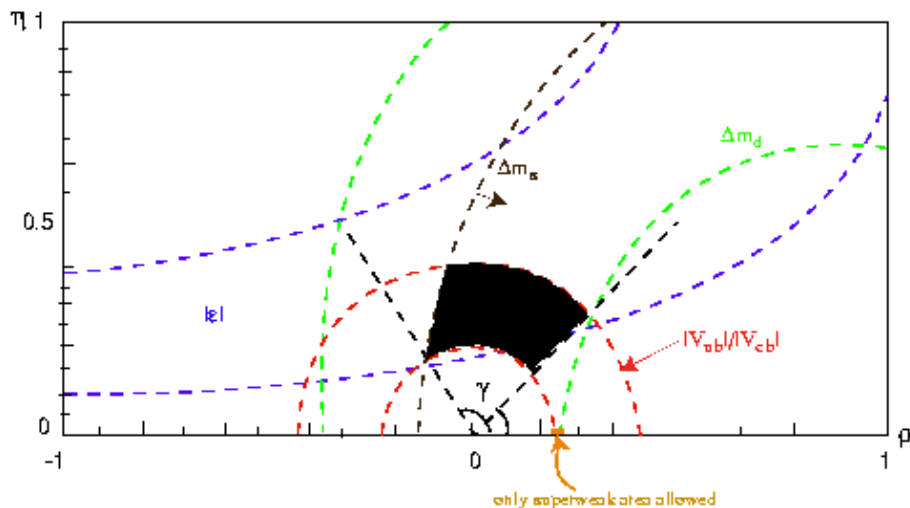


Figure 1: Allowed region in the $\rho - \eta$ plane. From Ref. [16]

CP-violating effect, which is independent of $|\epsilon|$. In the Kaon system this is provided by a measurement of $|\epsilon'/\epsilon|$ and, eventually, of the extremely rare process $K_L \rightarrow \pi^0 \nu \bar{\nu}$. For B-decays, the simplest additional CP-violating measurement involves extracting $\sin 2\beta$ from the study of $B_d \rightarrow \Psi K_S$ decays. I comment on each of these processes in what follows.

3.1 ϵ'/ϵ .

At the time of WIN99 one expected news any day from the two large experiments (KTeV and NA48) trying to measure ϵ'/ϵ . Both experiments have run in the last two years and their aim is to bring down the error for ϵ'/ϵ to a few times 10^{-4} . Hence, these experiments should be capable to resolve the issue of whether η is non-vanishing. Unfortunately, even with a measurement of ϵ'/ϵ it will be difficult to get a good estimate for η . As is well known,¹⁸ ϵ'/ϵ depends both on the contribution of gluonic Penguins and of electroweak Penguins. Although there is little uncertainty in the calculation of the coefficients of the relevant operators, since all attendant QCD effects have been computed to NLO,¹⁹ the hadronic matrix elements of the Penguin operators are

not well determined. Furthermore, it turns out that these two contributions tend to cancel, increasing the uncertainty of the theoretical predictions.

It is useful to illustrate this with an approximate formula^b given some time ago by Buras and Lautenbacher.²⁰ These authors write for $\text{Re } \epsilon'/\epsilon$ the expression

$$\text{Re } \epsilon'/\epsilon \simeq [36 \times 10^{-4}] A^2 \eta \left\{ B_6 - 0.175 \left(\frac{m_t^2}{M_W^2} \right)^{0.93} B_8 \right\} . \quad (5)$$

Here B_6 (B_8) are the contribution of the gluonic (electroweak) Penguins, appropriately normalized. Using the CKM analysis of Fig. 1, along with the measurement of $|V_{cb}|$ which fixes A , the parameter $A^2 \eta$ spans the range $0.18 \leq A^2 \eta \leq 0.31$. Whence one predicts

$$\text{Re } \epsilon'/\epsilon \simeq (6.5 - 11.3) \times 10^{-4} [B_6 - 0.75 B_8] . \quad (6)$$

In the vacuum insertion approximation $B_6 = B_8 = 1$. Lattice calculations²¹ and $1/N$ results¹⁹ give $B_6 = B_8 = 1 \pm 0.2$. That is, they reproduce the vacuum insertion result to 20%. Unfortunately, because of the cancellation between the gluonic and electromagnetic Penguin operators, the corresponding uncertainty for ϵ'/ϵ in Eq.(6) is much greater.

The experimental situation regarding ϵ'/ϵ was not clear at the time of WIN99, since the old CERN²² and FNAL²³ results for ϵ'/ϵ are mildly contradictory:

$$\text{Re } \epsilon'/\epsilon = \begin{cases} (23 \pm 3.6 \pm 5.4) \times 10^{-4} & \text{NA31} \\ (7.4 \pm 5.2 \pm 2.9) \times 10^{-4} & \text{E731} \end{cases} \quad (7)$$

However, naively, the theoretical formula (6) seems to favor the E731 result, since it appears much easier to get $\text{Re } \epsilon'/\epsilon \sim \text{few} \times 10^{-4}$ than a much larger number. Recently, however, the KTeV Collaboration announced a new result for ϵ'/ϵ obtained from an analysis of about 20% of the data collected in the last two years.⁵ Their result is

$$\text{Re } \epsilon'/\epsilon = (28 \pm 3 \pm 2.6 \pm 1) \times 10^{-4} = (28 \pm 4.1) \times 10^{-4}, \quad (8)$$

where the first error is statistical, the second is systematic and the third is an estimate of the error introduced by their Monte Carlo analysis. Strangely, this result is much closer to the old CERN result than the value obtained by E731,

^bFor a more accurate treatment, see the review of Buchalla *et al.*¹⁹

which was KTeV's precursor! More importantly, the value obtained is nearly 7σ away from zero, giving strong evidence that the CKM phase γ is indeed non-vanishing.

Although it is not possible to extract a good value for η from this measurement, it is clear that the superweak solution in Fig. 1 is now excluded by this new data. However, the rather large value for ϵ'/ϵ is theoretically surprising. To get a value for $\text{Re } \epsilon'/\epsilon$ as large as that of KTeV and NA48, either the cancellation between the gluonic and electroweak Penguins is highly ineffective and/or the overall size of B_6 and B_8 is much bigger than that suggested by the vacuum saturation approximation. Since $B_i \sim 1/m_s$ one can increase the overall size of these matrix elements by supposing that the strange quark mass m_s is much smaller than normally assumed. Whether this is warranted remains to be seen, although recent lattice estimates seem to point in this direction.²⁴

3.2 $K_L^0 \rightarrow \pi^0 \nu \bar{\nu}$.

The (extremely) rare decay $K_L^0 \rightarrow \pi^0 \nu \bar{\nu}$ provides a much cleaner measure of the CP-violating parameter η . Since $\pi^0 J^*$, with J^* a spin 1 virtual state, is CP odd, one can write the amplitude for $K_L \rightarrow \pi^0 J^*$ as²⁵

$$A(K_L \rightarrow \pi^0 J^*) = \epsilon A(K_1 \rightarrow \pi^0 J^*) + A(K_2 \rightarrow \pi^0 J^*), \quad (9)$$

where the second term is non-vanishing only if there is direct CP violation ($\eta \neq 0$). However, because of the factor of ϵ and the smallness of the CP-conserving decay $K_1 \rightarrow \pi^0 \nu \bar{\nu}$, to a very good approximation, $A(K_L \rightarrow \pi^0 \nu \bar{\nu}) \simeq A(K_2 \rightarrow \pi^0 \nu \bar{\nu})$. Furthermore, because the relevant K to π matrix element is well known from the corresponding charged decay, there is little theoretical uncertainty.

The direct CP-violating amplitude is dominated by loops involving top. Again, one can write an approximate formula for the branching ratio for this process, but now there is no matrix element uncertainty:²⁶

$$B(K_L \rightarrow \pi^0 \nu \bar{\nu}) = 8.07 \times 10^{-11} A^4 \eta^2 \left(\frac{m_t}{M_W} \right)^{2.3}. \quad (10)$$

A CKM analysis then leads to the prediction: $1.5 \times 10^{-11} < B(K_L \rightarrow \pi^0 \nu \bar{\nu}) \leq 4.4 \times 10^{-11}$. Unfortunately, this theoretical expectation is about four orders of magnitude below the present limit for this process, $B(K_L \rightarrow \pi^0 \nu \bar{\nu}) < 5.9 \times 10^{-7}$.²⁷ Given the difficulty of measuring this all neutral final state, it is

difficult to imagine trying to extract η this way. However, an experimental result would yield a value for η with very little theoretical error.

3.3 $\sin 2\beta$.

In my view, the measurement of $\sin 2\beta$ should provide, in the near term, the strongest confirmation of the basic correctness of the CKM paradigm. First of all, the CKM analysis of Fig. 1 implies a rather large value for $\sin 2\beta$, since β like γ is a large angle.^c Secondly, and of equal importance, $\sin 2\beta$ is accessible experimentally in a theoretically clear context. Let me briefly explain this last point.

As usual, to observe CP-violating effects one needs to have interference of two different amplitudes with different CP phases. This occurs naturally in the time evolution of B-decays. Consider specifically the decay of a state $B_d^{\text{phys}}(t)$ into some final state f . The state $B_d^{\text{phys}}(t)$ is defined by the property that at $t = 0$ it was a B_d state. Because of mixing, $B_d^{\text{phys}}(t)$ evolves in time as a linear superposition of B_d and \bar{B}_d states. Thus the decays of $B_d^{\text{phys}}(t)$ into the final state f can follow two different paths, and their interference can lead to CP-violating phenomena.

$B_d - \bar{B}_d$ mixing is dominated by the top quark box graph. As a result, the only large CP-violating phase which enters is the phase β of V_{td} . It is easy to show that, as a result of the mixing, one has

$$|B_d^{\text{phys}}(t)\rangle = e^{-iM_B t} e^{-\frac{\Gamma_B}{2}t} \left\{ \cos \frac{\Delta m_d}{2} t |B_d\rangle + i e^{-2i\beta} \sin \frac{\Delta m_d}{2} t |\bar{B}_d\rangle \right\} \quad (11)$$

It turns out that no other CP-violating phase will enter (in leading order in λ) if the underlying processes $B_d \rightarrow f$ and $\bar{B}_d \rightarrow f$ only involve the weak decays $b \rightarrow c\bar{c}s$ and $\bar{b} \rightarrow \bar{c}c\bar{s}$. This is obvious for the tree amplitudes, but it is also true for the $b \rightarrow s$ Penguin graph, since this is dominated by the top-loop and V_{ts} and V_{tb} are purely real. In addition, for decays of $B_d^{\text{phys}}(t)$ to CP self-conjugate states f ($\bar{f} = \eta_f f$, with $\eta_f = \pm 1$) the B_d and \bar{B}_d amplitudes are simply related. As a result, one finds for these decays a formula with no theoretical ambiguities at all, namely

$$\Gamma(B_d^{\text{phys}}(t) \rightarrow f) \Big|_{b \rightarrow c\bar{c}s} = \Gamma(B_d \rightarrow f) e^{-\Gamma_B t} \{1 + \eta_f \sin 2\beta \sin \Delta m_d t\} \quad (12)$$

^cFor instance, a recent CKM analysis¹⁵ gives $\sin 2\beta = 0.73 \pm 0.08$.

There is great deal of interest in measuring $\sin 2\beta$ through the study of the process $B_d^{\text{phys}}(t) \rightarrow \psi K_S$, as this process has both a large branching ratio and a nice signature. Of course, what is crucial for the measurement is being able to correctly tag the decay as originating from an initially produced B_d or \bar{B}_d state. Last summer, at the Vancouver Conference, both OPAL²⁸ and CDF²⁹ reported the first attempts at extracting $\sin 2\beta$. Soon after WIN99, CDF announced the result of an updated analysis of their data which, even with a large error, is already quite interesting: $\sin 2\beta = 0.79 \pm 0.44$.³⁰

A much more precise values for $\sin 2\beta$ should be forthcoming soon from the B factories at KEK and SLAC. In the B-factories one can measure $\sin 2\beta$ by looking at a variety of modes besides $B_d \rightarrow \psi K_S$ (e.g. $B_d \rightarrow \psi K_L$; $B_d \rightarrow D\bar{D}$). Of course, the precision with which each mode determines $\sin 2\beta$ differs, but consistency of the different results will provide an important cross check. A rather detailed analysis of the reach achievable with the BABAR detector at the SLAC B Factory is contained in the Babar Physics Book.¹⁶ For instance, using just the ψK_S and ψK_L modes, one expects to be able to measure $\sin 2\beta$ with an error $\delta \sin 2\beta = \pm 0.23$ ($\delta \sin 2\beta = \pm 0.09$) with an integrated luminosity of 5 fb^{-1} (30 fb^{-1}). This accuracy is of the order of the present uncertainty in $\sin 2\beta$ from a CKM analysis. However, the B-factory results will be an **actual measurements** of CP violation!

4 Successes and Challenges of Weak Decays.

As we alluded to in the introduction, weak decays of heavy quark systems are much more amenable to theoretical analysis. In these systems, a combination of a heavy quark expansion [HQET]⁴ and perturbative QCD permits rather precise predictions which, on the whole, have been well tested experimentally. In general, however, even here one has to restrict oneself to special theoretical regions or inclusive enough processes. For example, predictions for exclusive decays, like the process $B \rightarrow D^* \ell \bar{\nu}_\ell$, are valid only in the zero-recoil limit. Similarly, predictions for inclusive decays, which rely on parton-hadron duality, cease to be reliable near the kinematical limit. I will illustrate some of the theoretical issues with some examples.

4.1 Extracting $|V_{cb}|$ from Inclusive Semileptonic Decays.

Data on B semileptonic decays coming from the $\Upsilon(4s)$ and from Z decays are mildly inconsistent. As a result, the average width for $b \rightarrow c\ell\bar{\nu}_\ell$ decays [$\Gamma(b \rightarrow c\ell\bar{\nu}_\ell) = (66.5 \pm 3.0) \text{ ps}^{-1}$ ³¹] has still a 5% error. Since $\Gamma(b \rightarrow c\ell\bar{\nu}_\ell) \sim |V_{cb}|^2$ this experimental error implies a 2.5% experimental error on $|V_{cb}|$. This error is comparable to the theory error quoted in different recent analyses. ^{32,33} If the data from CLEO and LEP could be reconciled, potentially one should be able to reduce the experimental error above by a factor of 2. This raises the question of what accuracy one can hope to reach theoretically for $|V_{cb}|$?

The naive parton model formula for the B semileptonic width has a large uncertainty coming from the b -quark mass, since this enters raised to the 5th power, $\Gamma(b \rightarrow c\ell\bar{\nu}_\ell) \sim m_b^5$. This uncertainty is partially removed in the HQET. In this approach m_b is replaced by the B -quark mass M_B , and the parton model rate is modified by subleading $1/m_b$ corrections. These latter terms are proportional to matrix elements of local operators appearing in the operator product expansion of the tensor $T_{\mu\nu}$:

$$T_{\mu\nu} = \int d^4x e^{iqx} \langle B | \bar{b}(0) \gamma_\mu (1 - \gamma_5) c(x) \bar{c}(0) \gamma_\nu (1 - \gamma_5) b(0) | B \rangle, \quad (13)$$

whose imaginary part is related to the width Γ . ⁴ The upshot is that in HQET there are 2 new operators which enter the theory at $O(1/m_b^2)$, ^d whose matrix elements one needs to know:

$$\lambda_1 = \frac{1}{2M_B} \langle B | \bar{h}_b(iD)^2 h_b | B \rangle ; \quad \lambda_2 = \frac{g_3}{12M_B} \langle B | \bar{h}_b \sigma^{\mu\nu} G_{\mu\nu} h_b | B \rangle. \quad (14)$$

Here D and $G_{\mu\nu}$ are the usual covariant derivative and field strengths of QCD and $h_b(x) = (1 + \gamma \cdot v)/2 e^{im_b v \cdot x} b(x)$, where v^μ is the 4-velocity of the b -quark. In addition, the relation between m_b and M_b introduces a further parameter, $\bar{\Lambda}$, which, roughly speaking, accounts for the momentum distribution of the b -quark in the B -meson. One can write for the pseudoscalar and vector B -mesons the formulas ⁴

$$M_B = m_b + \bar{\Lambda} - \frac{\lambda_1 + \lambda_2}{2m_b} ; \quad M_B^* = m_b + \bar{\Lambda} - \frac{\lambda_1 - \lambda_2}{2m_b} \quad (15)$$

The hyperfine splitting between M_B^* and M_B serves to fix λ_2 and one finds $\lambda_2 \simeq 0.12 \text{ GeV}^2$. So, at this level of accuracy, there are only two free parameters left, λ_1 and $\bar{\Lambda}$.

^dOne can show that there are no $O(1/m_b)$ corrections appearing in the HQET.

Provided one has estimates for λ_1 and $\bar{\Lambda}$, the formula for the semi-leptonic width (including $O(1/m_b^2)$ terms and QCD corrections to $O(\alpha_s)^2$)³⁴ can be used to determine $|V_{cb}|$. One can extract values for these parameters by studying the moments of the hadronic mass spectrum³⁵

$$\langle (s_H - \bar{M}_B^2)^n \rangle = \frac{1}{\Gamma(B \rightarrow X_c \ell \bar{\nu}_\ell)} \int ds_H \frac{d\Gamma}{ds_H} (s_H - \bar{M}_B^2)^n, \quad (16)$$

where s_H is the invariant mass of the recoiling hadronic state and $\bar{M}_B^2 = \frac{1}{4}(M_B^2 + 3M_\ell^2)$. Similarly, moments of the electron energy spectrum³⁶ also depend on λ_1 and $\bar{\Lambda}$.

Recently, the CLEO Collaboration³⁷ has measured the first and second moments of the hadronic mass spectrum, obtaining for λ_1 and $\bar{\Lambda}$, the following values:

$$\bar{\Lambda} = (0.33 \pm 0.02 \pm 0.08) \text{ GeV}; \quad \lambda_1 = -(0.13 \pm 0.01 \pm 0.06) \text{ GeV}^2. \quad (17)$$

These results are similar to those obtained by Gremm *et al.*³⁸ by considering moments of the lepton spectrum integrated over a restricted energy range: $[\bar{\Lambda} = (0.39 \pm 0.11) \text{ GeV}; \lambda_1 = -(0.19 \pm 0.10) \text{ GeV}^2]$. Unfortunately, the situation is a bit confusing at the moment, since CLEO obtained a different set of values $[\bar{\Lambda} \simeq 1 \text{ GeV}; \lambda_1 \simeq -0.8 \text{ GeV}^2]$ ³⁷ by considering leptonic moments where one integrates over (nearly) all the lepton energy distribution. These preliminary results, however, may be an artifact since they are quite sensitive to corrections coming from the unmeasured pieces of the leptonic spectrum.

If one just uses the values of $\bar{\Lambda}$ and λ_1 from the hadronic energy spectrum analysis, then from the experimental value for the width $\Gamma(b \rightarrow c \ell \bar{\nu}_\ell)$, one deduces³⁹ that $|V_{cb}| = 0.0415 \pm 0.0010 \pm 0.0010$. So, indeed, the theoretical and experimental errors on $|V_{cb}|$ are comparable. Nevertheless, one should note³⁹ that if indeed $\bar{\Lambda} \simeq 1 \text{ GeV}$ and $\lambda_1 \simeq -0.8 \text{ GeV}^2$, then the value of $|V_{cb}|$ goes up by 7%! However, because the above value is quite consistent with that extracted from an exclusive analysis of the decay $B \rightarrow D^* \ell \bar{\nu}_\ell$ at zero recoil $[|V_{cb}| = 0.0387 \pm 0.0031]$ ⁴⁰, this suggests that the CLEO full lepton moment analysis is probably flawed.

4.2 Extracting V_{ub} from B -Decays.

Because the decay $b \rightarrow u \ell \bar{\nu}_\ell$ is highly suppressed relative to $b \rightarrow c \ell \bar{\nu}_\ell$, to extract $|V_{ub}|$ from exclusive decays one must focus on the limited kinematical

region where the decays to charmed states are forbidden. This restricted region is characterized by having the lepton energy near its upper end point $[E_\ell > (M_B^2 - M_D^2)/2M_D]$ and the produced hadronic mass squared $s_H < M_D^2$. Falk, Ligeti and Wise,⁴¹ have suggested that concentrating on the second kinematical restriction is better than just looking at the high energy end of the lepton spectrum. This is because for $s_H < M_D^2$ a continuum of states contribute and hadron-parton duality should be reliable. On the other hand, at the end of the lepton spectrum typically π and ρ exclusive states dominate.

The differential rate for B-decays into final states of a given hadronic mass squared s_H , for $\bar{\Lambda}M_B < s_H < M_D^2$, can be expressed in terms of a shape function $S(s_H, \bar{\Lambda})$ which is universal in character and depends on the b -quark momentum distribution parameter $\bar{\Lambda}$:⁴¹

$$\frac{d\Gamma}{ds_H} = \frac{G_F^2 M_B^3}{192\pi^3} |V_{ub}|^2 \left(1 - \frac{\bar{\Lambda}}{M_B}\right)^3 S[s_H, \bar{\Lambda}]. \quad (18)$$

If indeed $\bar{\Lambda} \sim 300 - 400$ MeV and its error can be kept to the level of $\delta\bar{\Lambda} \leq 50$ MeV, then it should be possible³⁹ to reduce the error on $|V_{ub}|$ to around 10%. This would be a considerable improvement over the present day exclusive determination of this matrix element from the decays $B \rightarrow \pi\ell\bar{\nu}_\ell$, $B \rightarrow \rho\ell\bar{\nu}_\ell$ [$|V_{ub}| = (3.3 \pm 0.2_{-0.4}^{+0.3} \pm 0.7) \times 10^{-4}$]⁴⁰ which contains a 20% model error.

4.3 $B \rightarrow X_s\gamma$.

Last year saw a refined measurement from CLEO⁴² (as well a first ALEPH result⁴³) on this important process. The branching ratio obtained $[\text{BR}(B \rightarrow X_s\gamma) = (3.15 \pm 0.35 \pm 0.32 \pm 0.26) \times 10^{-4}]$ includes an estimate of the error introduced by having to extrapolate to below $E_\gamma = 2.1$ GeV, where one cuts on the data. At the same time, a host of theoretical refinements became available bringing the theoretical expectations for $B \rightarrow X_s\gamma$ also into excellent shape. Thus a meaningful and stringent comparison between theory and experiment is now possible.

The QCD corrections to the effective Lagrangian describing this process

$$\mathcal{L}_{\text{eff}} = \frac{4G_F}{\sqrt{2}} V_{ts} V_{tb}^* c_7^{\text{eff}}(\mu) \frac{e}{16\pi^2} \bar{s}\sigma^{\nu\mu} m_s \frac{(1 + \gamma_5)}{2} b F_{\mu\nu} \quad (19)$$

are very important, but they are controllable and known. For example, the coefficient $c_7^{\text{eff}}(\mu) \simeq -0.19$, without taking into account of QCD. This number

changes by more than 30% when lowest order QCD effects are incorporated, and is quite μ -dependent. Fortunately, the full NLO QCD effects have now been calculated by a number of groups⁴⁴ leading to a stable result for $c_7^{\text{eff}}(\mu)$, with little μ -dependence. As a result, theoretically, the branching ratio for $B \rightarrow X_s \gamma$ is now known with an error of 10%—comparable to that of the CLEO result $[\text{BR}(B \rightarrow X_s \gamma)]_{\text{theory}} = (3.29 \pm 0.34) \times 10^{-4}$ ⁴⁵. Since this result is in excellent agreement with experiment, there is little room for beyond the Standard Model contributions.

The theoretical result for the $B \rightarrow X_s \gamma$ branching ratio is not that sensitive to $1/m_b^2$ corrections which, typically, are of $O(5\%)$.⁴⁶ However, as Kagan and Neubert⁴⁷ have pointed out, the differential branching ratio as a function of the photon energy is quite sensitive to the momentum distribution of the b quark in the B -meson. As a result, the photon energy spectrum in $B \rightarrow X_s \gamma$ can give information on the nonperturbative parameters $\bar{\Lambda}$ and λ_1 . This is apparent from the recent analysis of Neubert⁴⁵ which finds that the errors for the branching ratio for $B \rightarrow X_s \gamma$ due to $O(1/m_b^2)$ effects is about 10% if the cut on $(E_\gamma)_{\text{min}}$ is at 2.2 GeV, but is less than 5% when that cut is reduced to 2.0 GeV. More interestingly,^{45,48} one could imagine using the shape function obtained from analyzing the photon energy spectrum in $B \rightarrow X_s \gamma$ to extract from the differential rate for $B \rightarrow X_u \ell \bar{\nu}_\ell$ a more accurate value of $|V_{ub}|$.

5 Looking for the New CP-Violating Phases.

If the CKM model is correct, one expects the unitarity triangle to close, so that $\alpha + \beta + \gamma = \pi$. However, if there are other CP-violating phases arising from new physics, one can expect to alter this simple relation between the angles in the unitarity triangle. As we discussed earlier, the strongest predictions which emerge from a CKM analysis of the present data is that both the angles β and γ are rather large. Even so, it might well be that when one measures $\sin 2\beta$ with accuracy it will not agree with the value of $\sin 2\beta$ coming from the CKM analysis. In what follows, I want to briefly discuss how this might come about.

Recall from Eq. (11) that the phase 2β entered in the time evolution of $B_d^{\text{phys}}(t)$ as the CKM phase connected with $B_d - \bar{B}_d$ mixing. It is possible that other physics enters in this mixing beyond the CKM model, bringing additional CP-violation phases. Let us call the additional CP-violating phase entering in $B_d - \bar{B}_d$ mixing θ_M . Then, effectively, everywhere one should replace $\sin 2\beta \rightarrow \sin 2(\beta + \theta_M)$. Unfortunately, even pushing parameters to

extremes, it is difficult to generate a very large “new physics” CP-violating phase θ_M . For instance, in supersymmetric models at most $\theta_M \sim 10^\circ$.⁴⁹ Since the CKM β -range is $10^\circ \leq \beta \leq 27^\circ$, even such a large phase θ_M is difficult to detect!^e

More promising than the new mixing phases θ_M , are phases arising from new physics which may affect Penguin amplitudes. A good example is provided by the pure Penguin decay $b \rightarrow s\bar{s}s$. In the CKM model, the $b \rightarrow s$ Penguin amplitude is dominated by top loops and is purely real. However, in supersymmetry $\tilde{b} - \tilde{s}$ mixing can bring additional CP-violating phases and it is possible that the amplitude ratio $A(B_d \rightarrow \phi K_s)/A(\bar{B}_d \rightarrow \phi K_s) \equiv e^{i\Phi_p}$ reflect this “new” Penguin phase Φ_p . Because Penguin effects are subdominant in processes like $B_d \rightarrow \psi K_S$, the phase Φ_p is probably unimportant in this process. Hence the time evolution of $B_d^{\text{phys}}(t) \rightarrow \psi K_S$ essentially still measures $\sin 2\beta$. However, the time evolution of $B_d^{\text{phys}}(t) \rightarrow \phi K_S$ actually measures $\sin(2\beta + \Phi_p)$. So one can look for new physics CP-violating phases by comparing the values of the coefficient of $\sin \Delta m_d t$ in these two processes. Obviously, error control is crucial.

There are many other examples of such strategies. For instance, in the CKM model, the time evolution of $B_s^{\text{phys}}(t)$ into $\psi\phi$ should show no $\sin \Delta m_d t$ term, since there is no mixing phase for $B_s^{\text{phys}}(t)$ and there are no decay phases for $b \rightarrow c\bar{c}s$. Finding such a term could provide evidence for new physics. This discussion raises the issue of how well one can test the unitarity triangle by measuring directly, in addition to β , also α and γ . Let me consider each of these angles in turn.

5.1 α .

In principle, the angle α is measurable in an analogous way to β . One now needs to study the decays of $B_d^{\text{phys}}(t)$ into final states that can be accessed through a $b \rightarrow u$ transition. A good example is provided by $B_d \rightarrow \pi^+\pi^-$. If the quark decay $b \rightarrow u\bar{u}d$ is dominated by the tree amplitude, then

$$\frac{A(\bar{B}_d \rightarrow \pi^+\pi^-)}{A(B_d \rightarrow \pi^+\pi^-)} = \frac{A(b \rightarrow u\bar{u}d)}{A(\bar{b} \rightarrow \bar{u}ud)} \Big|_{\text{Tree}} = \frac{V_{ub}}{V_{ub}^*} = e^{-2i\gamma}. \quad (20)$$

^eNote that since the extra mixing phase θ_M obviously does not affect the CKM angle γ , one expects also that $\sin 2\alpha \rightarrow \sin 2(\alpha - \theta_M)$. However, α is even more uncertain than β and these effects are even harder to pin down.

This extra decay phase $e^{-2i\gamma}$ adds to the contribution from the $B_d - \bar{B}_d$ mixing phase $e^{-2i\beta}$. Using that $\alpha + \beta + \gamma = \pi$, one finds

$$\Gamma(B_d^{\text{phys}}(t) \rightarrow \pi^+\pi^-) \Big|_{\text{Tree}} = \Gamma(B_d \rightarrow \pi^+\pi^-) e^{-\Gamma_B t} \{1 - \sin 2\alpha \sin \Delta m_d t\} . \quad (21)$$

For $B_d \rightarrow \pi^+\pi^-$, however, one cannot neglect the effects of the Penguin graphs, since for $b \rightarrow u\bar{u}d$ decays these graphs have a different phase structure than the tree graphs.⁵⁰ While the tree graph phase is that of V_{ub} , $e^{-i\gamma}$, the $b \rightarrow d$ Penguin has a phase $e^{i\beta}$ —the phase of V_{td}^* entering in the dominant t -quark loop. Because these two phases are different, it is important to try to understand the effects of this “Penguin pollution”. Penguin pollution will alter Eq. (21) in two ways. Consider the parameter $\xi = e^{-2i\beta} A(b \rightarrow u\bar{u}d)/A(\bar{b} \rightarrow \bar{u}u\bar{d})$. Ignoring Penguins, ξ is simply $e^{-2i(\beta+\gamma)} = e^{2i\alpha}$. Including Penguins, ξ becomes

$$\xi = e^{2i\alpha} \left[\frac{1 + \frac{|P|}{|T|} e^{-i\alpha} e^{i\delta}}{1 + \frac{|P|}{|T|} e^{i\alpha} e^{i\delta}} \right] = |\xi| e^{2i\alpha_{\text{eff}}} \quad (22)$$

where δ is an (unknown) strong interaction phase. Because $|\xi|$ is not unity now, and $\alpha \neq \alpha_{\text{eff}}$, the rate for $B_d^{\text{phys}}(t) \rightarrow \pi^+\pi^-$ will now also have a $\cos \Delta m_d t$ term, as well as a modified $\sin \Delta m_d t$ term. It is easy to see that the rate formula becomes

$$\Gamma(B_d^{\text{phys}}(t) \rightarrow \pi^+\pi^-) = \Gamma(B_d \rightarrow \pi^+\pi^-) e^{-\Gamma_B t} \{1 + a_c \cos \Delta m_d t - a_s \sin \Delta m_d t\} \quad (23)$$

where

$$a_c = \frac{1 - |\xi|^2}{1 + |\xi|^2} ; \quad a_s = \frac{2|\xi|}{1 + |\xi|^2} \sin 2\alpha_{\text{eff}} . \quad (24)$$

Gronau and London⁵¹ suggested estimating Penguin pollution in the $B_d \rightarrow \pi\pi$ process through an isospin analysis. Their idea is simple to describe. If one neglects electroweak Penguins, then isospin is a good quantum number and one can use isospin to classify the various decay amplitudes. $A(B^+ \rightarrow \pi^+\pi^0)$ is a $\Delta I = 3/2$ amplitude, and as such must be purely given by tree graphs, since the Penguin graphs are $\Delta I = 1/2$. Because the phase of the tree graphs is that of V_{ub} , it follows that $A(B^+ \rightarrow \pi^+\pi^0) = e^{2i\gamma} A(B^- \rightarrow \pi^-\pi^0)$. Isospin, in addition, relates the decay modes of B_d, \bar{B}_d into $\pi\pi$ to the charged B decays:

$$\begin{aligned} \frac{1}{\sqrt{2}} A(B_d \rightarrow \pi^+\pi^-) + A(B_d \rightarrow \pi^0\pi^0) &= A(B^+ \rightarrow \pi^+\pi^0) \\ \frac{1}{\sqrt{2}} A(\bar{B}_d \rightarrow \pi^+\pi^-) + A(\bar{B}_d \rightarrow \pi^0\pi^0) &= A(B^- \rightarrow \pi^-\pi^0) . \end{aligned} \quad (25)$$

These expressions geometrically can be represented as two triangles in the complex plane, with a common base. It is easy to check that the misalignment angle between these triangles is related to the phase α_{eff} .⁵¹ Hence, from measurements of all the relevant rates one can infer the Penguin pollution.

There have been a number of model studies to see what kind of errors one might expect on α . One of the most complete of these studies is that done in the Babar Physics Book,¹⁶ where a variety of decay modes [$B_d \rightarrow \pi\pi, \rho\pi, \rho\rho, a_1\pi$] were considered. Because the relevant branching ratios are not known, some reasonable assumptions had to be made both for these quantities and to estimate the amount of Penguin pollution. Assuming an integrated luminosity of 30 fb^{-1} the resulting typical error expected for δa_s for the $\pi^+\pi^-$ mode was $\delta a_s \simeq 0.26$, while for the $\rho^0\rho^0$ mode this error was $\delta a_s \simeq 0.17$. Unfortunately, it is difficult to extrapolate from these results the expected error on $\sin 2\alpha$ since the connection of $\delta \sin 2\alpha$ to δa_s is itself channel- (and model-) dependent. Nevertheless, it appears difficult to imagine measuring $\sin 2\alpha$ to better than $\delta \sin 2\alpha = 0.2$.

5.2 γ .

The situation with the angle γ is perhaps even more challenging, but at the same time more interesting. A number of authors have suggested trying to extract γ by looking at various asymmetries in processes which are dominated by tree amplitudes, but where the final state is not CP self-conjugate. One suggestion⁵² is to study the time evolution of $B_s^{\text{phys}}(t) \rightarrow D\phi$, in which both the $\bar{b} \rightarrow \bar{u}c\bar{s}$ and $b \rightarrow c\bar{u}s$ processes participate. This process is sensitive to $\sin \gamma$, but it is very challenging experimentally both because it involves B_s mesons and because of the very rapid $B_s - \bar{B}_s$ oscillations. Bigi and Sanda, as well as Sachs,⁵³ suggested instead studying the time evolution of $B_d^{\text{phys}}(t)$ into $D^{*\pm}\pi^\mp$. Here the processes $\bar{b} \rightarrow \bar{u}c\bar{d}$ and $b \rightarrow c\bar{u}d$ are involved and the time evolution measures $\sin(2\beta + \gamma)$. In this later example, the experimental challenge is that the predicted effect is very small.

Alternatively, as suggested by Gronau and Wyler,⁵⁴ one can try to extract γ by using triangle relations involving charged B -decays, similar to those we discussed earlier for $B_d \rightarrow \pi\pi$. This is nicely illustrated by the processes $B \rightarrow DK$, although the effects involved are probably not measurable experimentally. Both the decays $B^+ \rightarrow \bar{D}^0 K^+$ and $B^- \rightarrow D^0 K^-$ are pure tree decays, involving $\bar{b} \rightarrow \bar{c}u\bar{s}$ and its complex conjugate. Since V_{cb} is real, it follows that $A(B^+ \rightarrow \bar{D}^0 K^+) = A(B^- \rightarrow D^0 K^-) = A_1$, where A_1 can be

taken as real by convention. On the other hand, the decays $B^+ \rightarrow D^0 K^+$ and $B^- \rightarrow \bar{D}^0 K^-$, which are governed by the tree process involving $\bar{b} \rightarrow \bar{u} c \bar{s}$ and its complex conjugate, involve V_{ub} and hence the phase γ . Hence one has

$$A(B^+ \rightarrow D^0 K^+) = A_2 e^{i\gamma} e^{i\delta} ; \quad A(B^- \rightarrow \bar{D}^0 K^-) = A_2 e^{-i\gamma} e^{i\delta} , \quad (26)$$

where δ is a strong rescattering phase. It is easy to see that by measuring the rates for two of the above processes, as well as the rates for $B^+ \rightarrow D_+^0 K^\pm$, where $D_+^0 = \frac{1}{\sqrt{2}}(D^0 + \bar{D}^0)$ is a CP eigenstate, one can reconstruct γ essentially by trigonometry.⁵⁴ Unfortunately, this will not work in practice because the triangles are too squashed. Furthermore,⁵⁵ these decays are affected by rescattering effects which further complicate matters.

One can apply these ideas to channels with bigger branching ratios. However, in general, now one has both tree and Penguin contributions. Perhaps one of the nicest examples is provided by $B \rightarrow \pi K$, where lots of interesting dynamical features appear.⁵⁶ I want to illustrate some of the issues involved in these decays by discussing the, so called, Fleischer-Mannel bound⁵⁷ on $\sin^2 \gamma$. If one retains only the gluonic Penguins and neglects altogether rescattering effects, then one has simple formulas for the decays $B^+ \rightarrow \pi^+ K^0$; $B_d \rightarrow \pi^- K^+$ and $\bar{B}_d \rightarrow \pi^+ K^-$. The first decay is purely a Penguin process and, because the $\bar{b} \rightarrow \bar{s}$ Penguin is dominated by the top quark, there is no CP phase. The other two decays involve both trees and Penguins, with the tree amplitude having the phase of V_{ub}^* or V_{ub} , respectively. Thus one can write $A(B^+ \rightarrow \pi^+ K^0) = P$, while $A(B_d \rightarrow \pi^- K^+) = -[P + T e^{i\gamma} e^{i\delta}]$ and $A(\bar{B}_d \rightarrow \pi^+ K^-) = -[P + T e^{-i\gamma} e^{i\delta}]$, where δ is an (uncalculable) strong rescattering phase between the Penguin and tree contributions.

Using the above, the Fleischer-Mannel ratio R is easily computed. One finds

$$R = \frac{\Gamma(B_d \rightarrow \pi^- K^+) + \Gamma(\bar{B}_d \rightarrow \pi^+ K^-)}{\Gamma(B^+ \rightarrow \pi^+ K^0) + \Gamma(B^- \rightarrow \pi^- \bar{K}^0)} = 1 + 2r \cos \delta \cos \gamma + r^2 \quad (27)$$

where $r = T/P$. If $R < 1$, there is clearly negative interference between the Penguin and tree amplitudes and one can get a bound on γ . Indeed, it is easy to show that this bound is: $R \geq \sin^2 \gamma$.⁵⁷ Present day data from CLEO⁵⁸ is tantalizing since it gives $R = 1.0 \pm 0.4 \pm 0.2 \pm 0.1$. However, even if the data were to get more precise, matters are not as simple because R receives important corrections both from electroweak Penguins and from rescattering effects.⁵⁹

Rescattering in the πK system can change $\pi^0 K^+$ into $\pi^+ K^0$. This, effectively, leads to the replacement of the Penguin amplitude P by ${}^{60} \tilde{P}[1 + \epsilon_a e^{i\gamma} e^{i\delta_a}]$. Here the parameter ϵ_a (and the strong interaction phase δ_a) are a measure of the rescattering and $e^{i\gamma}$ is the phase of V_{ub}^* . Electroweak Penguins have no weak CP phase, but introduce an additional strong interaction phase. Effectively they can be accounted for by the replacement: $T e^{\pm i\gamma} e^{i\delta} \rightarrow T e^{i\delta} [e^{\pm i\gamma} + q_{EW} e^{i\delta_{EW}}]$. Here δ_{EW} is another strong interaction phase, while q_{EW} parametrizes the strength of these contributions. These changes alter the Fleischer-Mannel bound to: $R \geq F(\epsilon_a; q_{EW}; \delta_a; \delta_{EW}) \sin^2 \gamma$, where F is a calculable function of these new parameters. Neubert ⁶⁰ has argued that the rescattering effects are small ($\epsilon_a \leq 0.1$), but that q_{EW} , in fact, can be large ($q_{EW} \sim 0.5$). If this is so, the Fleischer-Mannel bound is significantly affected. However, a somewhat different ratio studied by Neubert and Rosner ⁶¹ appears to be more robust.

Given the uncertainties in all the methods discussed, it is clear that it is difficult to estimate the accuracy one may ultimately obtain for γ . Nevertheless, because information on this angle can be obtained in a variety of ways, this may help narrow down a range for the CKM phase γ . Nevertheless, I remark that the extensive discussion presented in the Babar Physics Book ¹⁶ on γ only ended up by hazarding a guess on the accuracy which might be achieved. It is suggested there that, with lots of integrated luminosity (100 fb⁻¹), perhaps one could hope to determine γ to $\delta\gamma = \pm(10 - 20)^\circ$.

6 Concluding Remarks.

It is clear that much theoretical progress has been made to control uncertainties in the predictions for weak decays and CP-violation. Particularly for the B -system a combination of beyond the leading order QCD corrections and HQET, in specific and controlled circumstances, can give results with rather small theoretical errors. These results, in turn, allow for the extraction from the data of fundamental parameters, like the elements of the CKM matrix.

This said, however, one has to admit that our theoretical understanding of CP-violation is still very rudimentary. We have no real explanation of why $\bar{\theta} < 10^{-10}$, unless axions are really found; we also have no real clue if there are any other low-energy CP-violating phases besides the CKM phase γ —and even for γ our evidence is still rather tentative. Fortunately, we are at the threshold of a new era of experimentation. As we discussed, very recently KTeV announced a value for ϵ'/ϵ and this should be followed shortly by a

similar announcement from NA48. Furthermore, the Frascati Φ Factory and its detector KLOE should soon be producing data. On the B -decay front, CLEO keeps integrating luminosity and adding to our detailed knowledge of these decays. At the same time, very soon both the SLAC and KEK B -factories, with their detectors BABAR and BELLE, should be running providing new information on B CP-violation. The remarkable recent result on $\sin 2\beta$ from CDF argues that also the Tevatron, in its forthcoming run with the Main Injector, will contribute substantially in this area. So there is real hope that experiment will shed some clarifying light soon on the nature of CP-violation. Let us hope so!

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